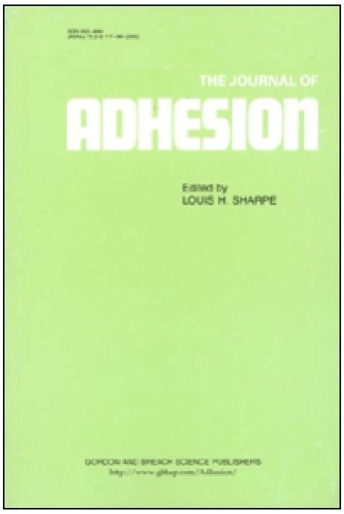

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Adhesion of a flat punch adhered to a thin pre-stressed membrane

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ADHESION OF A FLAT PUNCH ADHERED TO A THIN PRE-STRESSED MEMBRANE

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The mechanics of a circular membrane delaminating from a rigid punch is derived based on linear elasticity and an energy balance approach. Both the tensile prestress and the resulting concomitant stress upon external loading are considered. A "pull-off" phenomenon is predicted when the contact circle shrinks to a critical value between 0.1945 and 0.3679 of the film diameter, depending on the critical strain energy release rate and the prestress. These asymptotic limits match exactly with the prestress dominant model by Shanahan and prestress free model by Wan. Thin film delamination from a rectangular punch is also investigated. Unlike the circular punch, the rectangular contact is expected to reduce to a line contact with zero contact area at "pinch-off." The graphs and trends shown are useful in assessing thin film delamination assisted by a prestress.

Keywords: Adhesion; Membrane; Residual stress; Punch; Delamination

INTRODUCTION

Adhesion of thin membranes on rigid substrates is crucial in many aspects in electronics, micro electromechanical devices (MEMs), biology, and other fields. For instance, electronics packages fail when delamination or "popcorn" occurs, the operation of MEMs is obstructed when

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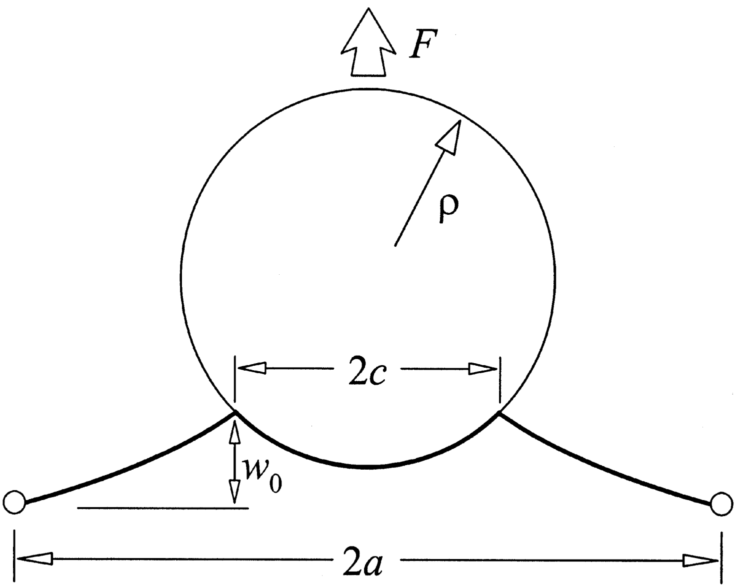
undesirable surface forces are present, and biological organs form when biocapsules such as cells adhere and aggregate. The presence of a tensile prestress or residual stress in the film (e.g., due to processing steps and thermal mismatch) will have a deleterious effect on adhesion.

Voluminous literature is available on techniques for measuring prestress in a thin flexible film and thin film adhesion on a rigid substrate. Williams summarized a large number of classical peel tests and blister tests and computed the strain energy release rates of prestressed films delaminating from rigid substrates [1]. Recently, other geometries were reported. For instance, Shanahan formulated the adhesion of a balloon onto a rigid substrate [2], as well as the adhesion of a rigid spherical cap to a circular film [3]; White considered the adhesion between a blistering film and a rigid plate [4]; Wan investigated the adherence between a thin film and an axisymmetric flat punch [5, 6] and a rectangular punch [7].

In this paper, we will first focus on the axisymmetric punch tests. Shanahan [3] considered a thin flexible *prestressed* membrane clamped at the perimeter and adhered to a punch with a spherical cap (Figure 1a). An external pulling force exerted on the punch caused delamination at the punch-film dissimilar interface. Based on linear elasticity and an energy balance, an unstable separation of the adherends was predicted when the contact circle shrank to $1/e \approx 0.3679$ of the film diameter, provided the radius of curvature of the punch is large compared with the ratio of film radius to thickness. Recently, Wan [5, 6] independently suggested a slightly different configuration (Figure 1b). It was a cylindrical punch with a flat end adhered to the film. The film was assumed free of residual stress, but it experienced a concomitant membrane stress due to application of external load and the subsequent change in film profile. Based on a similar energy balance method, spontaneous separation of the adherends, or "pull-off," was expected when the contact circle shrank to 0.1945 of the film diameter under fixed grip loading. In this paper, we attempt to construct a generalized thin film delamination mechanics to include the combined prestress and concomitant membrane stress and to cover wide ranges of the interfacial adhesion energy and prestress. We will show that the two existing theories are asymptotic limits of the generalized model.

A model for thin film delamination from a rectangular punch was constructed earlier and was experimentally verified [7]. Upon external loading, the rectangular contact of a prestress-free membrane shrinks to a line contact before complete separation of the adherends. In this article, a prestress is introduced to check if it has any effect on the delamination mechanics.



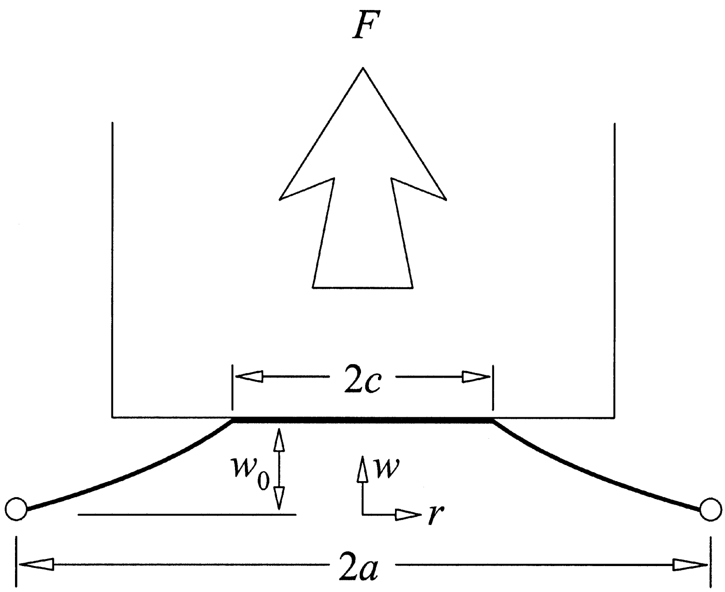
(a)

FIGURE 1 (a) Adherence between a punch with a spherical cap and a thin circular membrane clamped at the perimeter. In the limit of large radius of curvature of punch, the spherical cap becomes a flat cylindrical punch. (b) Adherence between a flat cylindrical punch and a thin circular membrane clamped at the perimeter. (Continued).

THEORY

Axisymmetric Flat Punch

We will start with an axisymmetric flat punch. Comparison with Shanahan's spherical cap solution will be discussed in the next section. Figure 1b shows a membrane with an elastic modulus, E, Poisson's ratio, ν, thickness, h, and radius, a, clamped at the perimeter and in adhesive contact with a rigid cylindrical punch with a radius, b ($\leq a$). The case where $b = a$ will first be considered. An external force, F, is applied quasi-statically to separate the adherends and drive a delamination along the punch-film interface. The contact radius, c, diminishes from an initial value of a and reaches equilibrium. The instantaneous membrane stress, N, initially equal to the pre-stress, N_0, is augmented by a concomitant stress, N_m, upon application of the external load, so that $N = N_0 + N_m$. Here the membrane stress is defined as $N = \sigma\ h$ having units of $\mathrm{N \cdot m^{-1}}$, where σ is the usual



(b)

FIGURE 1 Continued.

engineering stress having units of N·m^{-2}. The radial and tangential membrane stresses are assumed to be equal, and an *average* stress is acting on the membrane.* Equating the vertical forces requires

$$F = 2\pi r N \sin\theta \approx -2\pi r N \frac{dw}{dr} \tag{1}$$

which, after simple integration with respect to r, gives a membrane profile of

$$w = \frac{F}{2\pi N} \log\left(\frac{a}{r}\right) \tag{2}$$

The punch displacement is, therefore, given by

$$w_0 = \frac{F}{2\pi N} \log\left(\frac{1}{\zeta}\right) \tag{3}$$

*The exact von Karman equation of a circular plate can only be solved numerically. The radial and tangential stresses are in fact neither equal in magnitude nor spatially uniform. In order to derive an approximate analytical solution, equi-biaxial stress is assumed here. More details can be found in [6].

with $\zeta = c/a$. Since the elastic strain is given by $(\sec\theta - 1) \approx \theta^2/2$, the concomitant stress is shown to be [5, 6]

$$N_m = \frac{E'h}{2(a^2 - c^2)} \int_c^a \left(\frac{dw}{dr}\right)^2 r\,dr \tag{4}$$

where $E' = E/(1 - \nu)$. Eliminating N_m from Equations (3) and (4), the constitutive relation *without* delamination is found to be

$$F = \left[\frac{4\pi E'h}{(1 - \zeta^2)\log\zeta^2 \log\zeta^2}\right] w_0^3 + \left[\frac{4\pi N_0}{\log(1/\zeta^2)}\right] w_0 \tag{5}$$

The first ${w_0}^3$ term, corresponding to $N_m$only, is independent of N_0 and is consistent with our earlier work for $N_0 = 0$ [5]. The second w_0 term arises due to N_0 only. The total energy of the system is $U_T = U_P + U_S + U_E$, where $U_P = Fw_0$ is the potential energy due to the external load, $U_S = -(\pi c^2)\gamma$ is the surface energy with γ the critical strain energy release rate or the adhesion energy, and $U_E = -\int F dw_0$ is the elastic energy stored in the membrane annulus. Note that U_P is positively defined as it represents the energy input, while the negatively defined U_S and U_E are energy output. The mathematical formulation is consistent with classical linear elastic fracture mechanics [8]. Thus,

$$U_T = Fw_0 - \pi c^2\gamma - \left[\frac{4\pi E'h}{(1 - \zeta^2)\log\zeta^2 \log\zeta^2}\left(\frac{w_0^4}{4}\right) + \frac{4\pi N_0}{\log(1/\zeta^2)}\left(\frac{w_0^2}{2}\right)\right] \tag{6}$$

Note that the prestress is not released during delamination because of the constraints at the film's perimeter. A quasi-static equilibrium is reached when $dU_T/d(\pi c^2) = 0$, and Equation (6) leads to the energy balance:

$$\gamma = \left\{\frac{3(2 - 2\zeta^2 - \zeta^2\log\zeta^2)}{\zeta^2(1 - \zeta^2)^2[\log(1/\zeta^2)]^3}\left(\frac{E'h}{a^4}\right)\right\} w_0^4 + \left\{\frac{1}{\zeta^2[\log(1/\zeta^2)]^2}\left(\frac{1}{a^2}\right)\right\} 2N_0 w_0^2 \tag{7}$$

The first term is quadric in w_0 corresponding to the concomitant stress, while the second term is quadratic in w_0 corresponding to the prestress. To derive the constitutive relation $F(w_0)$ with delamination, both F and w_0 are expressed in terms of γ, N_0, and ζ. From Equations (5) and (7),

$$\omega_0 = \frac{1}{6}\sqrt{\frac{(1 - \zeta^2)\log\zeta^2}{2 - 2\zeta^2 - \zeta^2\log\zeta^2}\sqrt{\beta_0^2(1 - \zeta^2) - \sqrt{\Delta}}} \tag{8}$$

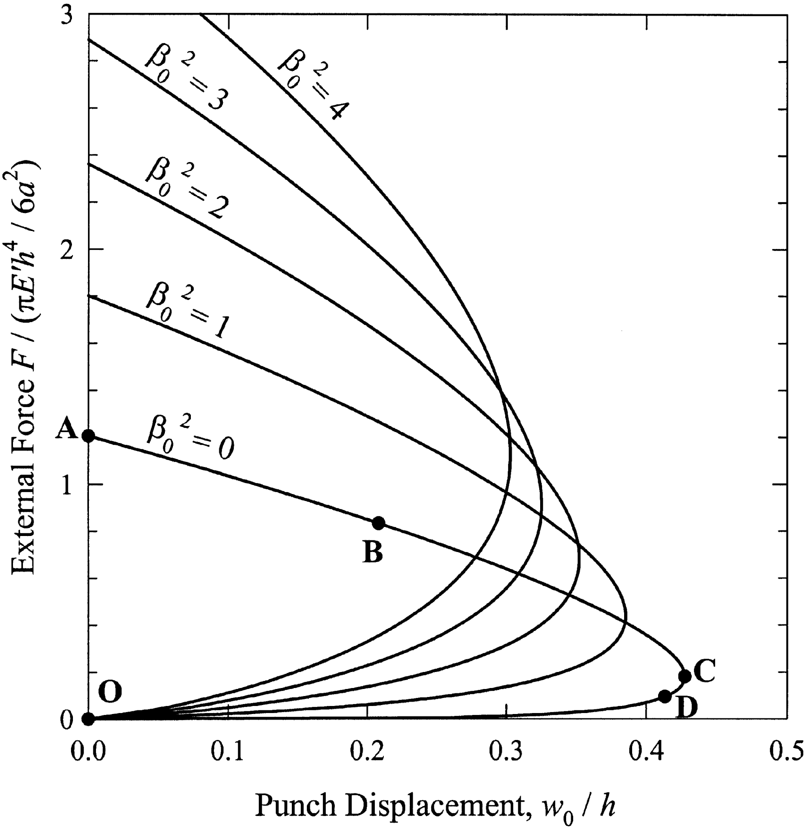
$$\varphi = \left[\frac{24}{(1-\zeta^2)(\log\zeta^2)^2}\right]\omega_0^3 - \left[\frac{1}{\log\zeta^2}\right]2\beta_0^2\omega_0 \tag{9}$$

where $\Delta = \beta_0^4(1-\zeta^2)^2 + \Gamma[144\zeta^2(1-\zeta^2)\log\zeta^2 + 72\zeta^4(\log\zeta^2)^2]$ and

$$\omega_0 = \left(\frac{1}{h}\right)w_0$$
$$\varphi = \left(\frac{6a^2}{\pi E' h^4}\right)F$$
$$\beta_0 = \left(\frac{12a^2}{E'h^3}\right)^{1/2}\sqrt{N_0}$$
$$\Gamma = \left(\frac{6a^4}{E'h^5}\right)\gamma$$

By eliminating ζ from Equations (8) and (9), $\varphi(\omega_0)$ or $F(w_0)$ can be found in an analytical form. To avoid involved mathematics, $\varphi(\omega_0)$ is cast as a parametric function with a varying ζ.

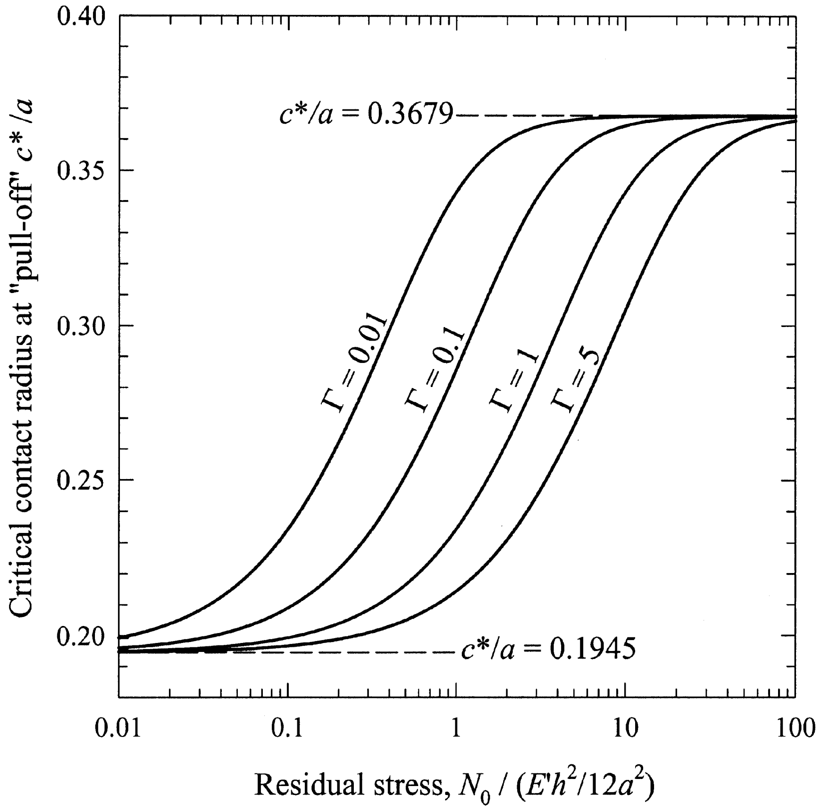
Figure 2a shows a family of $F(w_0)$ with delamination for a fixed γ and varying N_0. For the case of $N_0 = 0$ (or $\beta_0 = 0$), there are two alternative modes of external loading: fixed load (force-controlled) and fixed grip (displacement-controlled). The fixed load configuration follows a trajectory OA. Starting from point O where $F = 0$, the punch remains in full contact with the membrane until F reaches a threshold F_{th} at point A. Further increase beyond F drives a spontaneous delamination through the entire interface separating the adherends. The fixed grip configuration follows a different trajectory OABCDO. A force of F_{th} is needed to initiate delamination at point A. Increase in w_0 leads to a stable delamination along ABC and a contraction of the contact circle, while the external force diminishes to maintain equilibrium. A "pull-off" instability occurs at point C when $(dw_0/dF) = 0$ or $(dF/dw_0) \to \infty$, and $c^* = 0.1945a$. The asterisk superscript hereafter denotes "pull-off". Further increase beyond w_0^* causes a catastrophic delamination through the punch-film interface. The branch CDO satisfies the energy balance but is physically inaccessible. This is consistent with our earlier work for $N_0 = 0$ [5, 6] and is here extended to $N_0 > 0$. Similar pull-off behavior is predicted for non-zero N_0 as shown in Figure 2a. Figure 2b shows a family of the pull-off radius, c^*, as a function of N_0 for various γ (or Γ). Note that (c^*/a) is bounded by a lower limit of 0.1945 and an upper limit of $1/e \approx 0.3679$, with a transition around $\gamma \approx (2h^2/a^2)\,N_0$. When N_0 is small compared with N_m, c^* tends to the lower limit as expected. When N_0 is large, the energy stored in the elastic medium is already large and is augmented



(a)

FIGURE 2 (a) Constitutive relations for fixed adhesion energy $\Gamma = (6a^4/E'h^5)\gamma = 1$ and residual stress $\beta_0^2 = (12a^2/E'h^3)N_0 = 0, 1, 2, 3,$ and 4. Delamination under fixed grip follows the trajectory OABCDO, where points A, B, C, and D correspond to $(c/a) = 1.00$, 0.70, 0.1945, and 0.12, respectively. Pull-off occurs at point A in a fixed load configuration and at point C in a fixed grip configuration. Curve OCD denotes a nonphysical branch of the energy balance equation. (b) Critical contact radius, c^*, at the onset of unstable delamination as a function of N_0 for $\Gamma = (6a^4/E'h^5)\gamma = 1$. (Continued).

further by the concomitant stress during loading. Energy balance thus requires a *premature* pull-off with a larger c^* that approaches the upper limit. An intermediate N_0 involves a competition between N_0 and N_m and, thus, c^* goes through a transition. Figure 3a shows $F(w_0)$ for a fixed N_0 and varying γ. The trajectory for fixed grip loading follows curve OABCDO for $(6a^4/E'h^5)\gamma = 1$, and $(12a^2/E'h^3)N_0 = 1$.

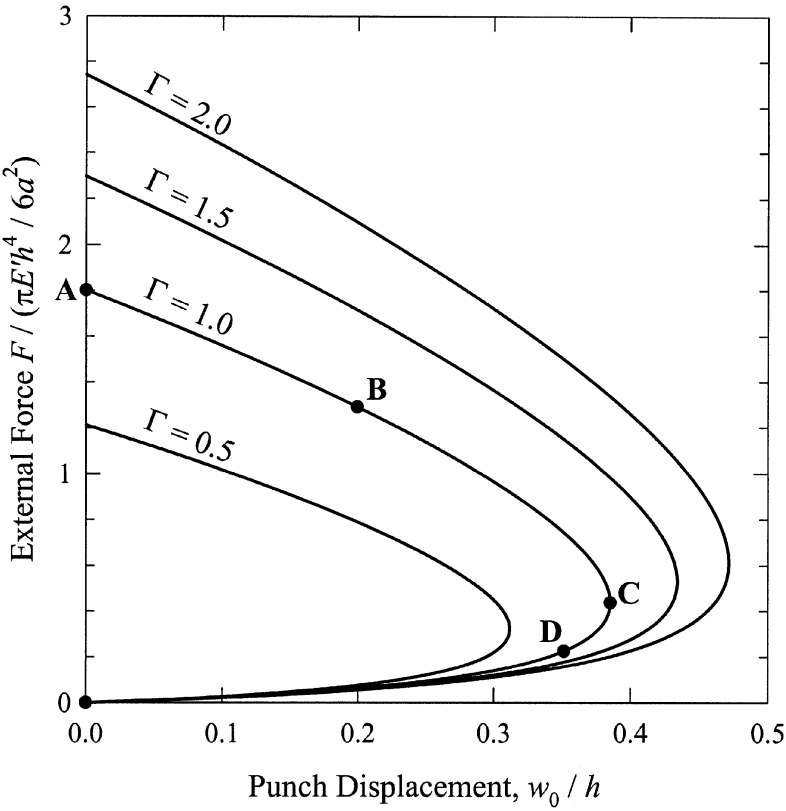


(b)

FIGURE 2 Continued.

All curves exhibit the same "pull-off" characteristics. Increase in γ pushes w_0^* to a larger value since more strain energy is needed to cause delamination in a strong interface. Figure 3b shows a family of c^* as a function of γ for various N_0. The ratio (c^*/a) is again bounded by 0.3679 and 0.1945. When N_0 dominates in weak interfaces (small γ), c^* approaches the upper limit.

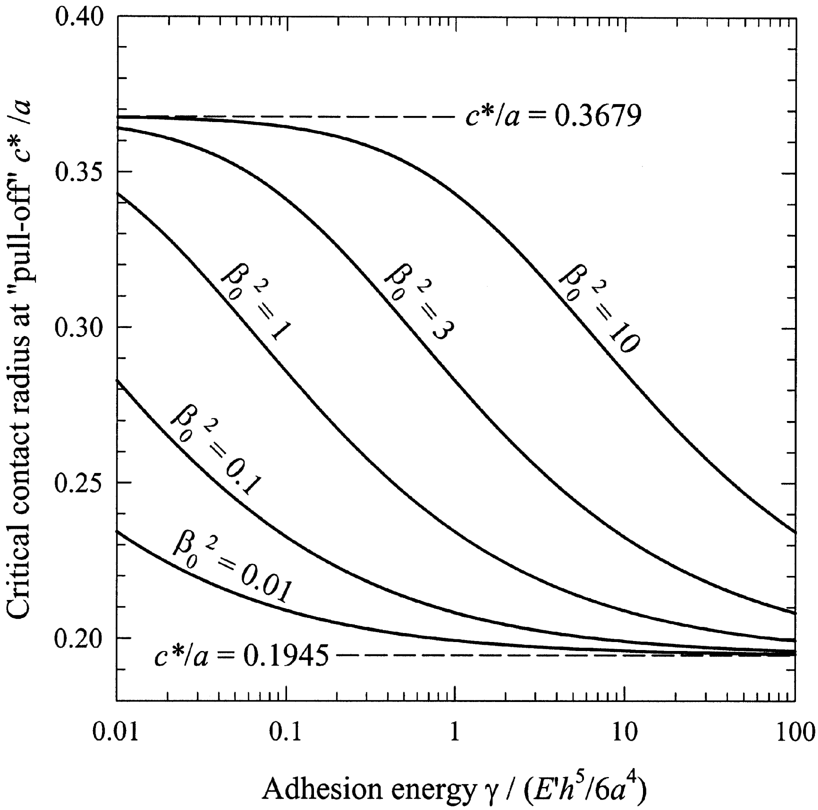
The punch radius is hitherto taken to be equal to that of the film $(b = a)$. For the case of $b < a$, initial punch movement is accommodated by stretching of the annular membrane rather than an intermediate interfacial delamination. The constitutive relation in Equation (5) is to be followed until the onset of delamination. As an illustration, Figure 4a shows the mechanical response for $b > c^*$ with $b = 0.80a$, $c^* = 0.22a$, $(6a^4/E'h^5)\gamma = 1$, and $(12a^2/E'h^3)N_0 = 1$. Curve OAB



(a)

FIGURE 3 (a) Constitutive relations for fixed residual stress $\beta_0^2 = (12a^2/E'h^3)N_0 = 1$ and adhesion energy $\Gamma = (6a^4/E'h^5)\gamma = 0.5, 1.0, 1.5,$ and 2.0. Delamination follows the trajectory OABCDO, where points A, B, C, and D correspond to $(c/a) = 1.00$, 0.70, 0.2343, and 0.12, respectively. Pull-off occurs at point A in a fixed load configuration and at point C in a fixed grip configuration. (b) Critical contact radius, c^*, at the onset of unstable delamination as a function of γ for $\beta_0^2 = (12a^2/E'h^3)N_0 = 1$. (Continued).

denotes the constitutive relation without delamination and is a monotonic increasing function of w_0. Curve O'BCDO represents the energy balance with OD being the nonphysical branch. Starting from zero punch displacement at point O, increase in w_0 causes the non-contact annulus to strain while the punch remains in full contact with the membrane along OAB. Point B represents the onset of delamination that continues via point C until pull-off at point D. Spontaneous separation of the punch from the film follows and the load drops to



(b)

FIGURE 3 Continued.

zero at point E. Figure 4b shows two trajectories OAB and ODE for $b \leq c^*$ with the same γ and N_0. Curve OAB represents the case where $b = c^* = 0.22a$. Here only a gradual straining of the membrane annulus occurs, but there is no stable delamination. Pull-off occurs at point B, the intersection of the constitutive equation and the energy balance curve. The external force drops to zero at point C. Curve ODE represents the case where $b = 0.12\ a < c^*$. Again, no stable delamination is allowed. Pull-off occurs at point E on the nonphysical branch of the energy balance.

Rectangular Punch

Figure 5 shows a sketch of a rectangular punch adhered to a membrane. The clamps, instead of being a ring, are two infinitely long bars

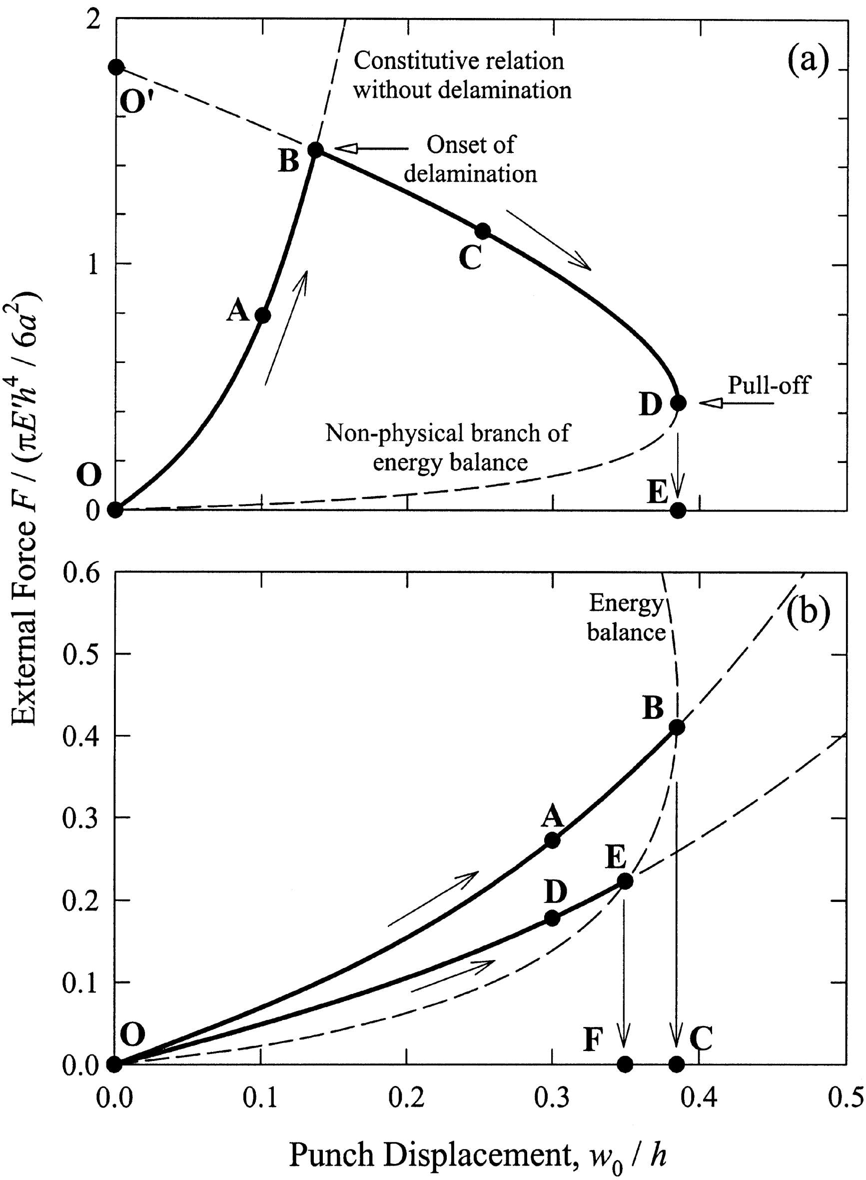


FIGURE 4 Mechanical response for $\Gamma = (6a^4/E'h^5)\gamma = 1$, $\beta_0^2 = (12a^2/E'h^3)$ $N_0 = 1$, and $c^* = 0.22a$. (a) For $b = 0.80a$, the trajectory follows the curve OABCD. (b) For $b = 0.22a$, trajectory follows OAB; for $b = 0.12a$, trajectory follows ODE.

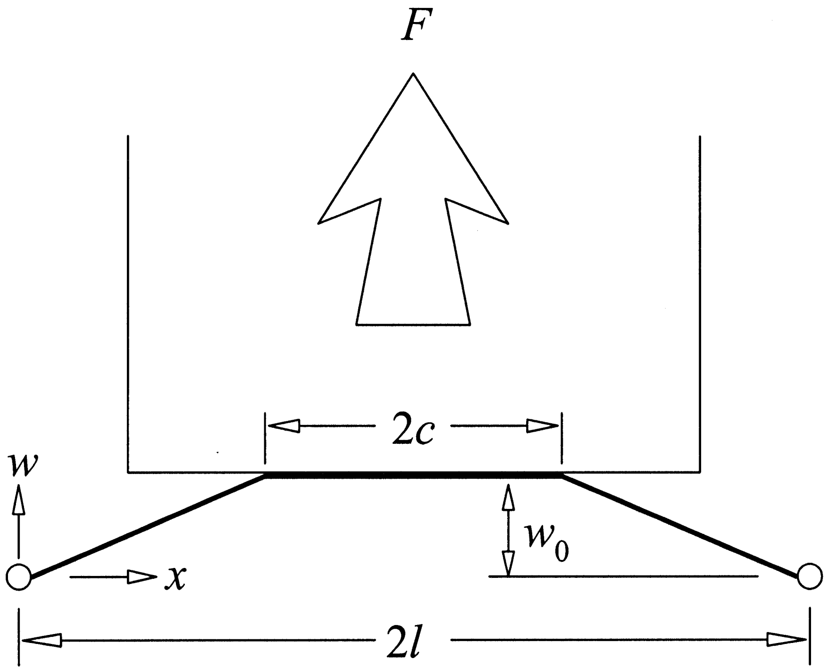


FIGURE 5 Adherence between a flat rectangular punch and a thin rectangular membrane.

holding the two opposite ends of the rectangular film. The film has a width of $2l$ and a rectangular contact width of $2c$. The external force per unit breadth is F. The delamination mechanics are formulated in a similar manner as in the previous subsection and our previous work [7]. In a 1-D configuration, the stretching membrane stress is in fact uniform and no approximation is needed as for the axisymmetric geometry. The membrane profile is linear and is given by

$$w = \left(\frac{F}{2N}\right)x \tag{10}$$

The punch displacement is therefore given by

$$w_0 = \left(\frac{Fl}{2N}\right)\lambda \tag{11}$$

with $\lambda = 1 - c/l$. The exact concomitant membrane stress is given by

$$N_m = \frac{E'h}{2\lambda l}\int_0^{l-c}\left(\frac{dw}{dx}\right)^2 dx \tag{12}$$

Eliminating N_m from Equations (11) and (12), the constitutive relation without delamination is derived:

$$F = \left(\frac{E'h}{\lambda^3 l^3}\right) w_0^3 + \left(\frac{2N_0}{\lambda l}\right) w_0 \qquad (13)$$

The first cubic term in w_0 is independent of N_0, consistent with our earlier work for $N_0 = 0$ [7], while the second linear term in w_0 describes the effect due to prestress. The total energy of the system is derived and the energy balance equation becomes

$$\gamma = \left\{\frac{1}{\lambda^4}\left(\frac{3E'h}{8l^4}\right)\right\} w_0^4 + \left\{\frac{1}{\lambda^2}\left(\frac{1}{4l^2}\right)\right\} 2N_0 w_0^2 \qquad (14)$$

The first quadric term and second quadratic term in w_0 correspond to the concomitant stress and the prestress, respectively. It can be shown from Equations (13) and (14) that

$$\frac{\omega_0}{\lambda} = \frac{1}{\sqrt{18}} \sqrt{-\beta_{0r}^2 + \sqrt{\beta_{0r}^2 + 72\Gamma_r}} \qquad (15)$$

$$\varphi_r = 6\left(\frac{\omega_0}{\lambda}\right)^3 + \beta_{0r}^2\left(\frac{\omega_0}{\lambda}\right) \qquad (16)$$

with

$$\omega_0 = \frac{w_0}{h}$$

$$\varphi_r = \left(\frac{6l^3}{E'h^4}\right) F$$

$$\beta_{0r} = \left(\frac{12l^2}{E'h^3}\right)^{1/2} \sqrt{N_0}$$

$$\Gamma_r = \left(\frac{6l^4}{E'h^5}\right) \gamma$$

From Equations (15) and (16), it is obvious that both φ_r and (ω_0/λ) are functions of β_{0r} and Γ_r only. Therefore, once γ and N_0 are fixed, both F and (w_0/λ) are fixed. Figure 6 shows $F(w_0)$ for $\gamma = E'h^5/6l^4$, $N_0 = 0$, and $b = 0.8l$. Curve OA denotes the constitutive relation without delamination, and curve AB denotes the energy balance. The path OABC is followed no matter whether the external loading is applied under fixed load or fixed grip. The initial stage of loading will traverse along path OA with no delamination until $F = F^*$ at point A.

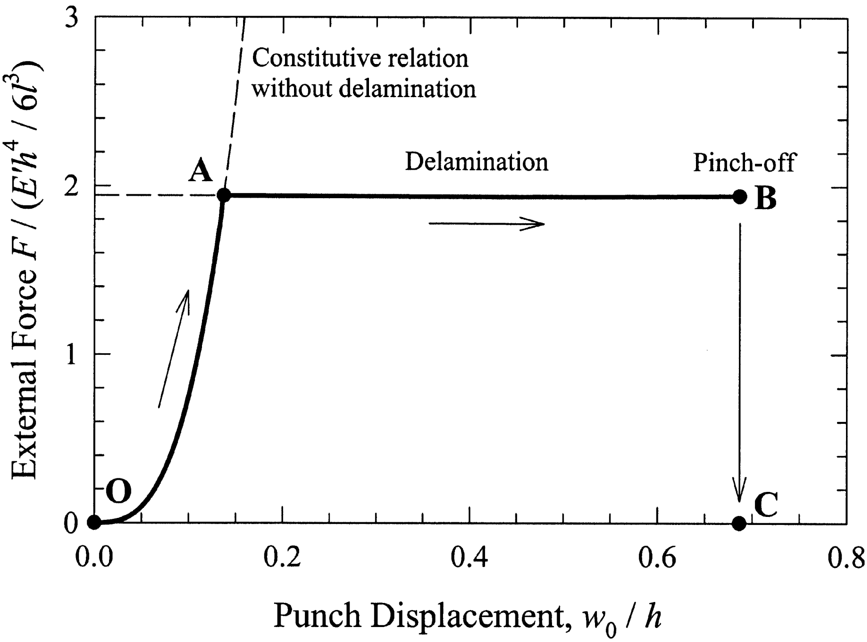


FIGURE 6 Mechanical response for $\Gamma = (6l^4/E'h^5)\gamma = 1$, $\beta_0^2 = N_0 = 0$, and $b = 0.80l$. External loading under either fixed load or fixed grip leads to trajectory OABC.

If F maintains at this level under fixed load, delamination will continue in a neutral equilibrium fashion until complete separation of the adherends at point B. If w_0 increases further from point A, a steady delamination directly proportional to w_0 continues until point B. Note that no abrupt pull-off with a nonzero contact is expected, but a "pinch-off" with an ultimate line contact is expected instead. This is consistent with our previous prediction for $N_0 = 0$ and is here extended to cases where $N_0 > 0$.

DISCUSSION

The presence of a nonzero "pull-off" radius in our punch-film model is remarkable. The classical Johnson-Kendall-Roberts (JKR) theory of adhesion between rigid solid bodies shows a similar "pull-off" event at finite contact radius [9]. The present new method offers an excellent opportunity for experimentalists measuring thin film adhesion, especially when the external force is extremely small and difficult to monitor. The prestress can be measured by the contact radius at pull-off, since it is manifested by any positive deviation from the lower limit of $c^* = 0.1945$. To measure the prestress and elastic modulus of the membrane, the same loading geometry can be adopted provided that no delamination occurs, and this is known as the shaft loaded blister test [10, 11]. The adhesion energy can thus be determined by c^* and $w_0{}^*$.

It is interesting to compare the above theory with Shanahan's punch model [3] (Figure 1a).[§] A spherical cap of radius, ρ, adheres to a flexible membrane under a prestress of N_0. The concomitant stress is neglected so that the effective membrane stress remains constant at N_0 upon external load. At equilibrium, the cap apex is at a distance δ from the plane of the nondeformed membrane. The profile of the noncontact annulus is found to be

$$w = k \log\left(\frac{a}{r}\right) \tag{17}$$

with an implicit proportionality constant k. The constitutive relation is found to be

$$\delta = \frac{\log(1/\zeta^2)}{4\pi N_0} F - \frac{a^2}{2\rho}\left(1 + \frac{a^2}{4\rho^2}\right) \tag{18}$$

To calculate γ, $\rho \gg c$ is assumed such that the contact circle is virtually planar with an area πa^2, and the surface energy U_S becomes $-(\pi a^2)\gamma$. The other energy terms—U_P, U_E, and U_T—are derived, and the identical energy balance approach as above is adopted to obtain γ:

$$\gamma = \frac{N_0 \delta}{2c[\log(a/c)]^2}\left[\frac{\delta}{c} + \frac{c}{\rho}(1 + 2\log(a/c))\right] \tag{19}$$

The delamination mechanics are similar to our model, in that, once the punch is raised from the film, delamination occurs until instability sets in. Stability of the configuration is determined by $(\partial\gamma/\partial c)$, where a negative quantity denotes an unstable equilibrium and catastrophic delamination growth while a positive quantity leads to a stable quasi-static equilibrium. It can be easily shown from Equation (19) that $(\partial\gamma/\partial c) < 0$ requires $(c/a) < (1/e) \approx 0.3679$, such that pull-off occurs once the circle shrinks to 0.3679 of the film diameter.

[§]The names given to variables used in this section are different from Shanahan's original paper [3], but the mathematical expressions are equivalent.

In our model of a flat punch, the contact circle is planar rather than being a spherical cap, yet the delamination mechanics should approach Shanahan's model in the limit of $\rho \gg c$ and $\delta \approx w_0$. Several remarks are noted as follows:

1. Shanahan foresaw the necessity of including the concomitant stress, N_m, as discussed in his paper [3] but did not attempt to compute it explicitly and assumed $N_m \ll N_0$ and $N \approx N_0$ throughout. One consequence of such an assumption is the difficulty in defining the constant k in Equation (17). Our model explicitly gives $k = (F/2\pi N)$. Thus, in case of zero residual stress ($N \approx N_0 = 0$), k becomes undefined in Shanahan's model.
2. Neglecting the concomitant stress, N_m, in Shanahan's model leads to significant error, especially when N_0 is small and N_m becomes the dominant membrane stress. The consequences can be seen in the constitutive relation in Equation (18), the strain energy release rate in Equation (19), and the pull-off contact radius. The constitutive relation in Equation (18) reduces to

$$F = \left[\frac{4\pi N_0}{\log(1/\zeta^2)}\right] w_0 \tag{20}$$

in the flat punch limit, which is identical to the linear w_0 term in Equation (5). The ignored ${w_0}^3$ term in Equation (5) becomes dominant at small N_0. The strain energy release rate in Equation (19) reduces to

$$\gamma = \left\{\frac{1}{\zeta^2[\log(1/\zeta^2)]^2}\right\} \frac{2N_0 w_0^2}{a^2} \tag{21}$$

in the flat punch limit, which is identical to Equation (7) if the ${w_0}^4$ term in Equation (7) is ignored. The pull-off contact radius is predicted by Shanahan to be always $(c^*/a) = 0.3679$ in the presence of any non-zero residual stress. The fact that $0.1945 \leq (c^*/a) \leq 0.3679$ at pull-off as predicted by our model shows that Shanahan's model is valid only for large N_0.
3. Shanahan suggested that the critical force at pull-off was related to γ, N_0, ρ, and a, even though the exact analytical form was thought to be too mathematically involved to be computed. We provide a procedure in the present work to compute the numerical pull-off force and relate it to other quantities.
4. The present model can easily be modified to accommodate the spherical cap geometry by replacing all w_0 in Equations (5) and (7)

by $\delta + c^2/2\rho$. Such changes do not lead to any notable consequences in the delamination mechanics.

Our generalized model, albeit spanning Shanahan's predominant prestress limit and Wan's negligible prestress limit, experiences similar shortcomings as discussed by Shanahan [3] and Wan [5, 6]. For instance, the viscoelastic behavior of a polymer film, which is not accounted for, could lead to a zero pull-off contact radius in practice; the exact von Karman solution that violates the uniform membrane stress assumption in this paper could lead to some minor errors in the pull-off parameters; the neglected mode II fracture, dissipative mechanisms in delamination, and finite range of intersurface forces could have an effect on the delamination mechanics.

CONCLUSION

The significance of attractive surface forces between a thin, flexible, prestressed membrane and a rigid substrate has been demonstrated. It is shown for a circular membrane delaminating from a rigid punch that complete separation of the adherends occurs when the contact circle reduces to a critical nonzero dimension. The critical ratio of radii of the contact circle to the membrane depends only on the adhesion energy of the dissimilar interface and the magnitude of the prestress or residual stress of the membrane, but it is *independent* of the stiffness and thickness of the film. The mechanical properties and geometry of the film are only manifested by the critical external load and punch displacement at pull-off. The axisymmetric punch test is particularly appealing to the experimentalists when measuring residual stress and adhesion energy, especially when the external force is too small to be accurately calibrated.

We have also derived the mechanics of a thin film delaminating from a rectangular punch. Unlike the axisymmetric geometry, the 1-D configuration requires the contact area to decrease to zero at complete separation of the adherends. The ultimate line contact is independent of the prestress and adhesion energy. It is easy to deduce that for an elliptical punch, as long as the delamination front complies with a proportional reduction of the planar elliptical geometry, the pull-off contact area should be finite and lies somewhere between that of the axisymmetric and rectangular counterparts.

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